MATH

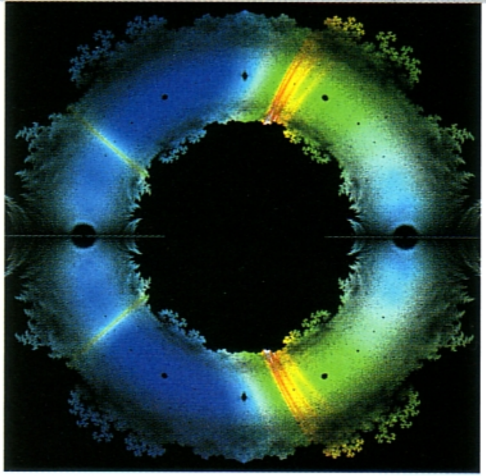
A Digital Slice of Pi

THE NEW WAY TO DO PURE MATH: EXPERIMENTALLY BY W. WAYT GIBBS

"One of the greatest ironies of the information technology revolution is that while the computer was conceived and born in the field of pure mathematics, through the genius of giants such as John von Neumann and Alan Turing, until recently this marvelous technology had only a minor impact within the field that gave it birth." So begins *Experimentation in Mathematics,* a book by Jonathan M. Borwein and David H. Bailey due out in September that documents how all that has begun to change. Computers, once looked on by mathematical researchers with disdain as mere calculators, have gained enough power to enable an entirely new way to make fundamental discoveries: by running experiments and observing what happens.

The first clear evidence of this shift emerged in 1996. Bailey, who is chief technologist at the National Energy Research Scientific Computing Center in Berkeley, Calif., and several colleagues developed a computer program that could uncover integer relations among long chains of real numbers. It was a problem that had long vexed mathematicians. Euclid discovered the first integer relation scheme—a way to work out the greatest common divisor of any two integers—around 300 B.C. But it wasn't until 1977 that Helaman Ferguson and Rodney W. Forcade at last found a method to detect relations among an arbitrarily large set of numbers. Building on that work, in 1995 Bailey's group turned its computers loose on some of the fundamental constants of math, such as log 2 and pi.

To the researchers' great surprise, after months of calculations the machines came up with novel formulas for these and other nat-



COMPUTER RENDERINGS of mathematical constructs can reveal hidden structure. The bands of color that appear in this plot of all solutions to a certain class of polynomials (specifically, those of the form $\pm 1 \pm x \pm x^2 \pm x^3 \pm \ldots \pm x^n = 0$, up to $n = 18$) have yet to be explained by conventional analysis.

ural constants. And the new formulas made it possible to calculate any digit of pi or log 2 without having to know any of the preceding digits, a feat assumed for millennia to be impossible.

There are hardly any practical uses for such an algorithm. A Japanese team used it to check very rapidly a much slower supercomputer calculation of the first 1.2 trillion digits of pi, completed last December. A pickup group of amateurs incorporated it into a widely distributed program that let them tease out the quadrillionth digit of pi. But mathematicians, stunned by the discovery, began looking hard at what else experimentation could do for them.

Recently, for example, the mathematical empiricists have advanced on a deeper question about pi: whether or not it is normal. The constant is clearly normal in the conventional sense of belonging to a common class. Pi is a transcendental number—its digits run on forever, and it cannot be expressed as a fraction of integers (such as $^{355}/_{113}$) or as the solution to an algebraic equation (such as $x^2 - 2 = 0$). In the universe of all known numbers, transcendental numbers are in the majority.

But to mathematicians, the "normality" of pi means that the infinite stream of digits that follow 3.14159... must be truly random, in the sense that the digit 1 is there exactly one tenth of the time, 22 appears one hundredth of the time, and so on. No particular string of digits should be overrepresented, whether pi is expressed in decimal, binary or any other base.

Empirically that seems true, not only for pi but for almost all transcendental numbers. "Yet we have had no ability to prove that even a single natural constant is normal," laments Borwein, who directs the Center for Experimental and Constructive Mathematics at Simon Fraser University in British Columbia.

"It now appears that this formula for pi found by the computer program may be the key that unlocks that door," Bailey says. He and Richard E. Crandall of Reed College have shown that the algorithm links the normality problem to other, more tractable areas of mathematics, such as chaos theory and pseudorandom number theory. Solve these related (and easier) problems, and you prove that pi is normal. "That would open the floodgates to a variety of results in number theory that have eluded researchers for centuries," Borwein predicts.

CRUNCHING NUMBERS

Mathematical experiments require software that can manipulate numbers thousands of digits long. David H. Bailey has written a program that can do math with arbitrary precision. That and the PSLQ algorithm that uncovered a new formula for pi are available at www.nersc.gov/~dhbailey/mpdist/

A volunteer effort is under way to verify the famous Riemann Hypothesis by using distributed computer software to search for the zeros of the Riemann zeta function. (German mathematician Bernhard Riemann hypothesized in 1859 that all the nontrivial zeros of the function fall on a particular line. See "Math's Most Wanted," Reviews, on page 94.) To date, more than 5,000 participating computers have found more than 300 billion zeros. For more information, visit www.zetagrid.net